Your MP3 Player and OFDM Modulator: About D-to-A Conversion

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Overview

- D/A conversion
- Introduction to OFDM
- Review of non-IDFT-based signal model and its PSD
  - ✓ Non-IDFT-based signal model for OFDM system and PSD
  - ✓ OFDM signal model in IEEE 801.11.a
  - ✓ OFDM signal model in IEEE 801.16.e and WiBro
- IDFT-based signal model and its PSD
  - ✓ IDFT-based discrete-time signal model
  - ✓ IDFT-based continuous-time signal model
  - ✓ New insight of PSD for OFDM subcarriers
- Conclusions

D/A Conversion

❖ Audible ranges of acoustic signals

- Human: 20Hz ~ 20 kHz (85 Hz ~ 1100 Hz)
- Dog: 15 Hz ~ 50 kHz (452 Hz ~ 1080 Hz)
- Frog: 50 Hz ~ 10 kHz (50 Hz ~ 8 kHz)
- Cat: 60 Hz ~ 65 kHz (760 Hz ~ 1520 Hz)
- Grasshopper: 100 Hz ~ 15 kHz (7 kHz ~ 100 kHz)
- Dolphin: 150 Hz ~ 150 kHz
- Bat: 1 kHz ~ 120 kHz

❖ About what you are familiar with

- .WAV: uncompressed, various sampling rates
- CD: uncompressed, error-control coded, 44.1 kHz sampling rate, 20 kHz signal, 2 channels
- .mp3: compressed (psychoacoustic compression), CD quality to FM radio quality

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D/A Conversion

❖ Psychoacoustic Compression for MP3

- Thresholding sounds below the hearing range
- Frequency masking
- Temporal masking
- Elimination of low frequency stereo information

❖ What kind of processing do we need to convert the stored digital signal to a continuous electrical signal that drives the speaker?

D/A Conversion

❖ D/A conversion

Digital signal	$\xrightarrow{\text{D/A conversion}}$	Continuous signal
$x[n]$		$y(t)$

❖ Interpolation understood in time domain

Converted continuous time signal $y(t)$ is given by

$$y(t) = \sum_{n=-\infty}^{\infty} x[n]p(t - nT)$$

$1/T$: conversion rate

$p(t)$: interpolation function

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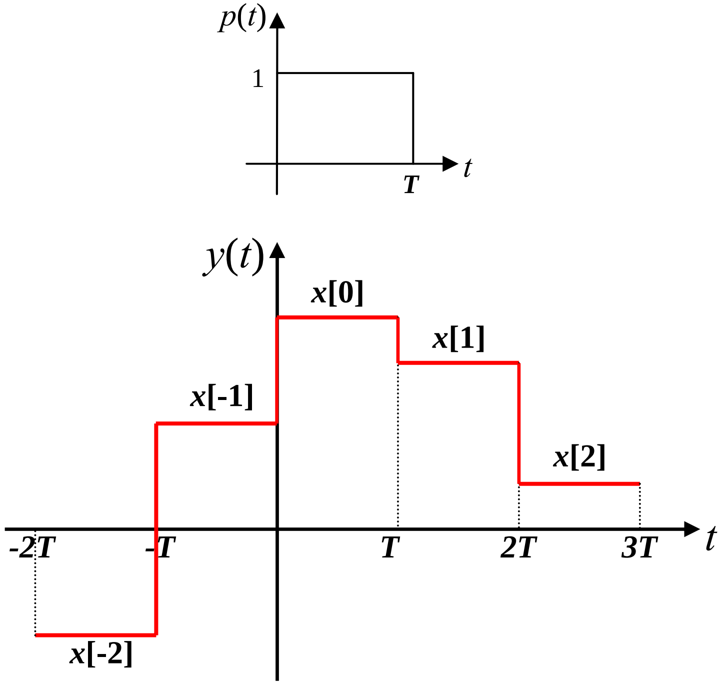
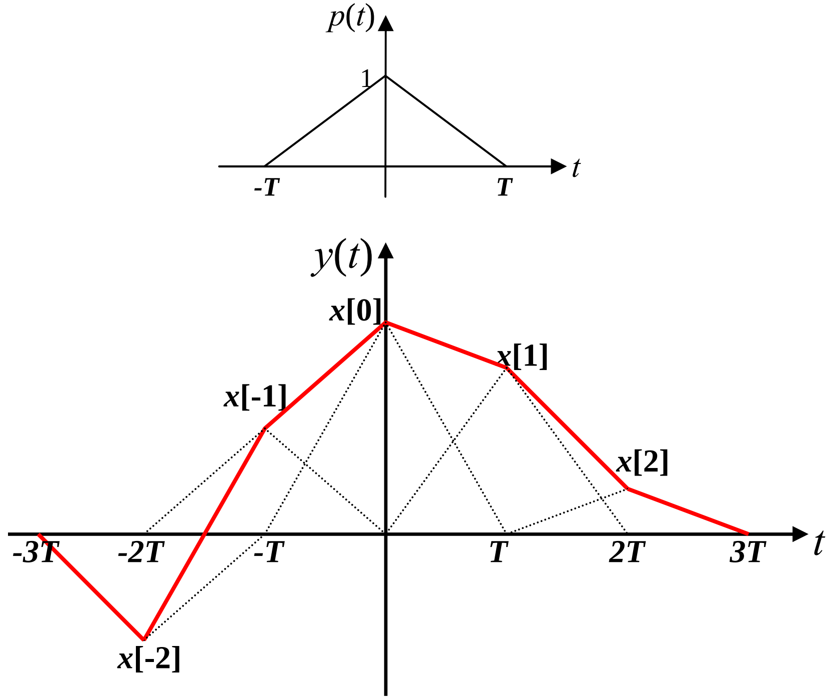
D/A Conversion (cont.)

❖ Interpolation understood in time domain (cont.)

[Example 1] Rectangular interpolation func.



[Example 2] Triangular interpolation func.



In general, a practical interpolation function may be more complex than the above examples.

By Nyquist theorem, sinc pulse makes the perfect reconstruction of the original signal.

D/A Conversion (cont.)

❖ Interpolation understood in frequency domain

Q. When $y(t)=\sum_{n=\infty}^{-\infty} x[n]p(t-nT)$, find $Y_c(f)$ in terms of $X_d(f)$, T, and $P(f)$.

A. $\boxed{Y_c(f)=P(f)X_d(fT)}$

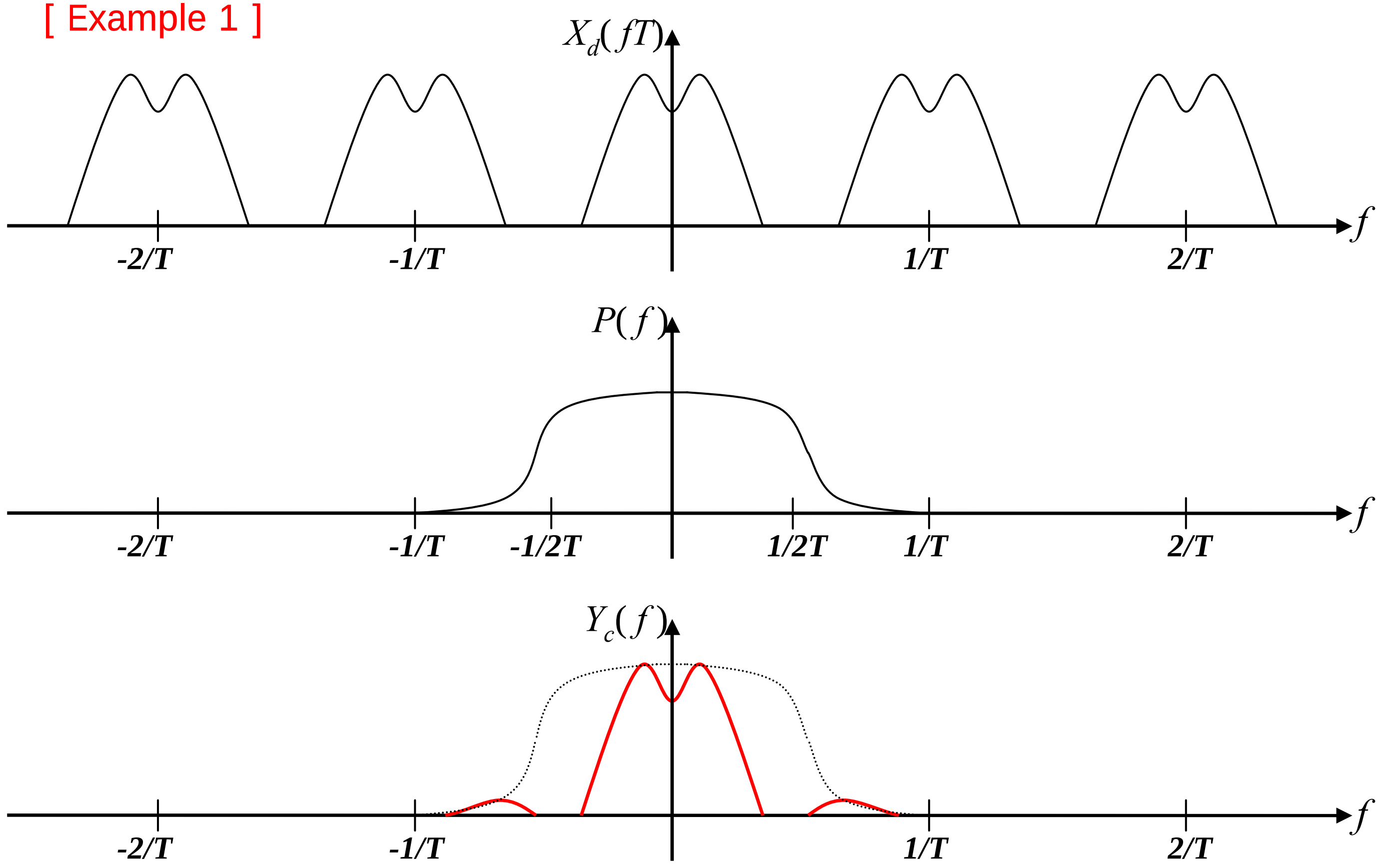
(Sol)

$$
\begin{aligned}
Y_c(f) &= \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,\mathrm{dt} \\
&= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x[n]p(t-nT)e^{-j2\pi ft}\,\mathrm{dt} \\
&= \sum_{n=-\infty}^{\infty} x[n]\int_{-\infty}^{\infty} p(t-nT)e^{-j2\pi ft}\,\mathrm{dt} \\
&= \sum_{n=-\infty}^{\infty} x[n]P(f)e^{-j2\pi fnT} \\
&= P(f)\sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi fnT} \\
&= P(f)X_d(fT)
\end{aligned}
$$

D/A Conversion (cont.)

❖ Interpolation understood in frequency domain (cont.)

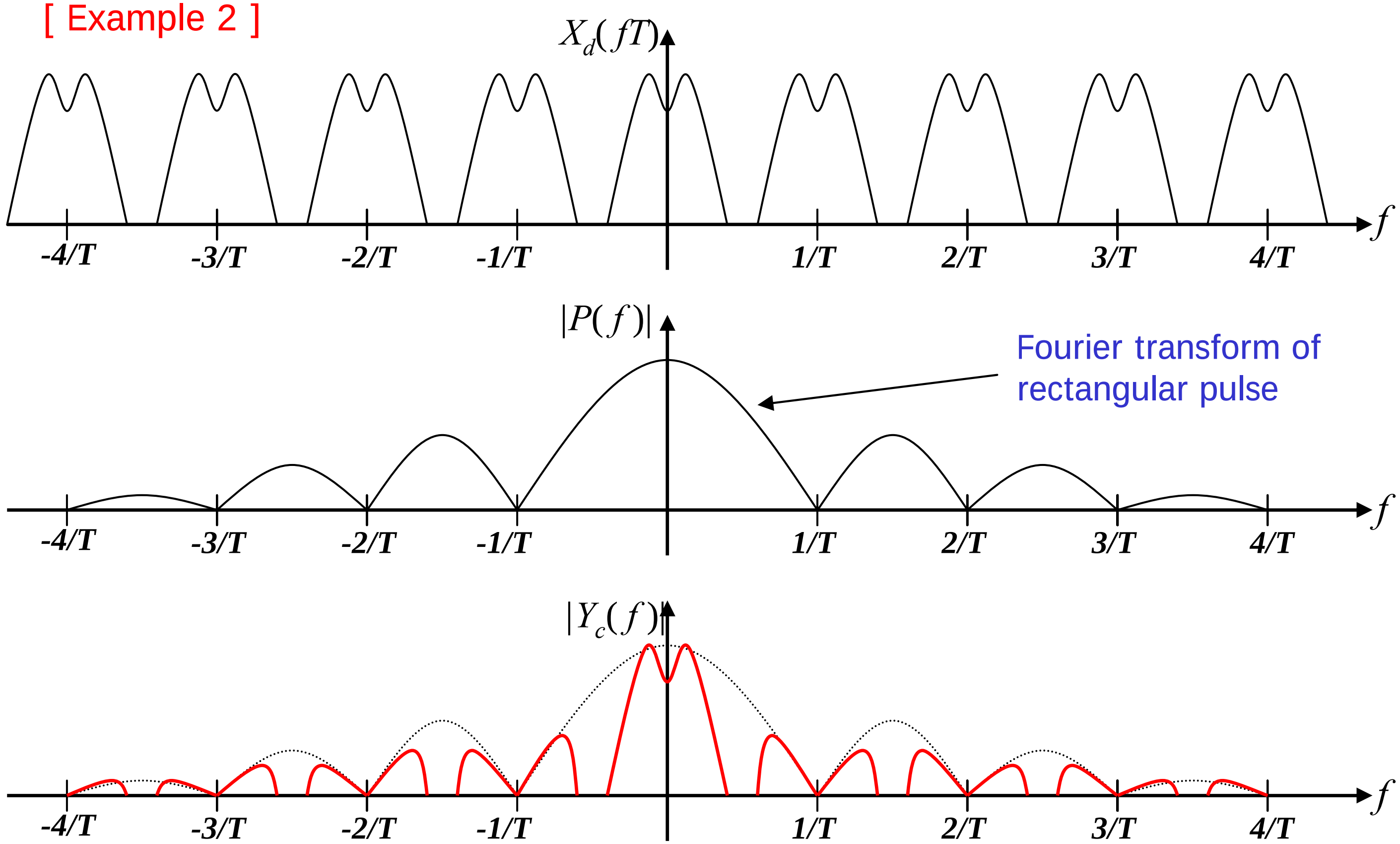
[Example 1]



D/A Conversion (cont.)

❖ Interpolation understood in frequency domain (cont.)

[Example 2]

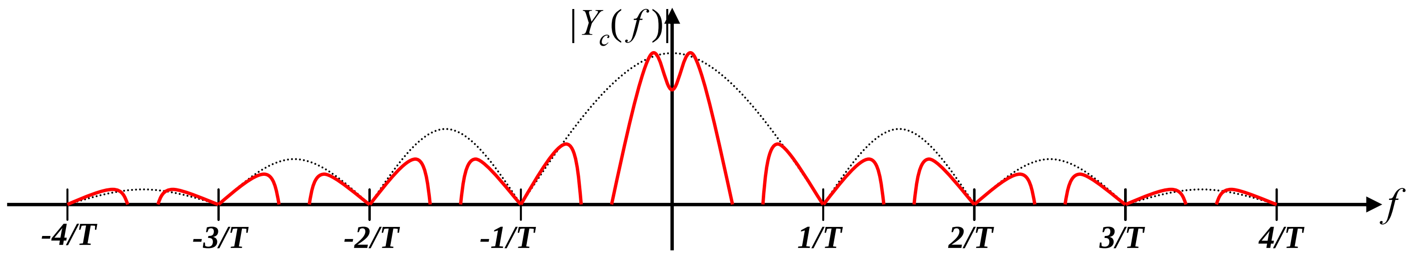


By using the rectangular pulse interpolation, the high frequency components are made.

D/A Conversion (cont.)

❖ Interpolation understood in frequency domain (cont.)

[Example 3 : WAV file]



(1) Audio file extracted from a movie

Sampling rate : 4kHz / Mono / 8bit

Good interpolation *Bad interpolation*

"Your Honor, every one of these letters is addressed to Santa Claus. The Post Office has delivered them. Therefore the Post Office Department, a branch of the Federal Government, recognizes this man Kris Kringle to be the one and only Santa Claus." - [*John Payne from Miracle On 34th Street* (1947)]

(2) Sinusoid freq is swept as a ramp function

Sampling rate : 4kHz / Mono / 8bit

Good interpolation *Bad interpolation*

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D/A Conversion (cont.)

- D/A converter with pre-distortion

Q. Suppose that we want to generate $y(t) = \sum_{n=-\infty}^{\infty} x[n]p(t-nT)$.

However, if we only have an interpolator with $1/T$ and $g(t)$, what can we do?

A. We filter $x[n]$ to $\tilde{x}[n]$ such a way that $\tilde{X}_d(fT)Q(f) = P(f)X_d(fT)$.

$\Longleftrightarrow$ We need to design a discrete time filter $h[n]$ such that

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow \tilde{x}[n]$$

with $\tilde{X}_d(fT) = X_d(fT)H(fT) = X_d(fT)\dfrac{P(f)}{Q(f)}$

However, $\dfrac{P(f)}{Q(f)}$ may not be easy to design.

➔ Usually a DAC upsamples the input discrete time signal and processes it.

D/A Conversion (cont.)

- Upsampling

Original signal	Upsampling	Upsampled signal
$\ldots, x[-1],\ x[0],\ x[1], \ldots$	$\longrightarrow$	$\ldots, x[-1],\ \underbrace{0, \ldots, 0}_{N\text{-1 zeros}}, x[0], \underbrace{0, \ldots, 0}_{N\text{-1 zeros}}, x[1], \ldots$

Q. Let $y[n] = x[n/N]$ for n=integer multiple of N, and 0, otherwise

Find $Y_d(f)$ in terms of $Y_d(f)$ and N.

A. $\boxed{Y_d(f) = X_d(fN)}$

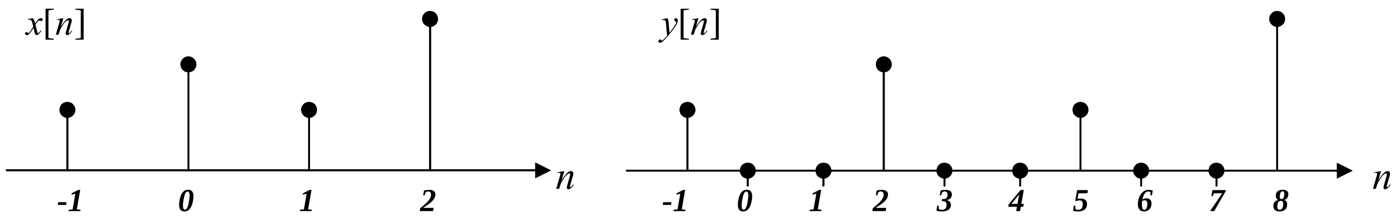
(Sol)
$$Y_d(f) = \sum_{n=-\infty}^{\infty} y[n] e^{-j2\pi fn}$$
$$= \sum_{m=-\infty}^{\infty} x[m] e^{-j2\pi fNm}$$
$$= X_d(fN)$$

D/A Conversion (cont.)

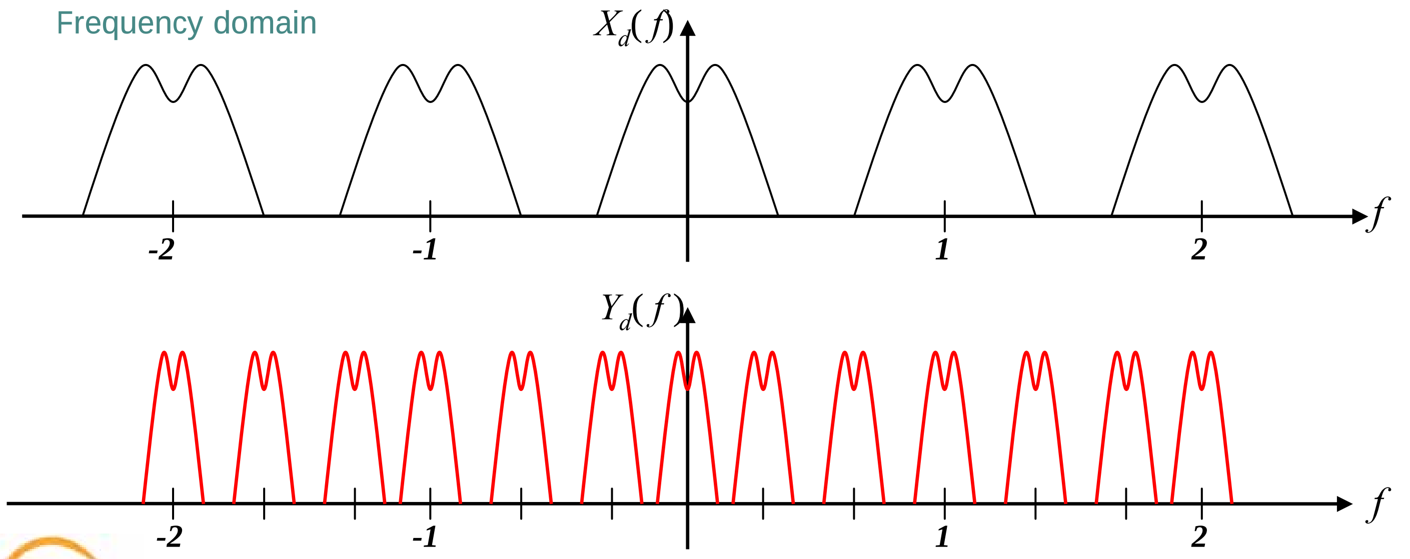
- Upsampling (cont.)

[Example : 3X upsampling with zero insertion]

Time domain



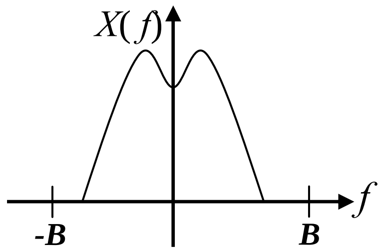
Frequency domain



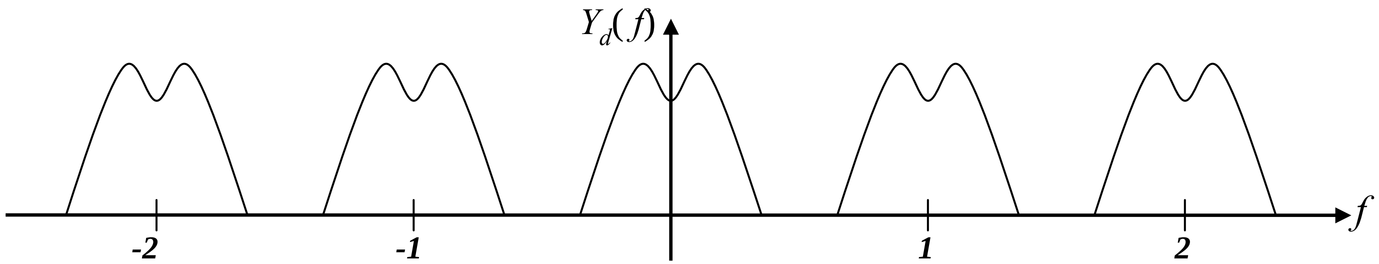
D/A Conversion (cont.)

❖ Examples for Upsampling

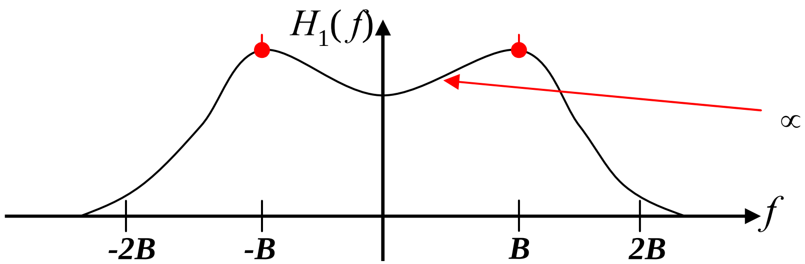
We want to generate a continuous-time signal $x(t)$ having



Assume：A discrete-time system that generates a discrete-time signal $y[n]$ is given.



Assume：A continuous-time system $h_1(t)$ is given.

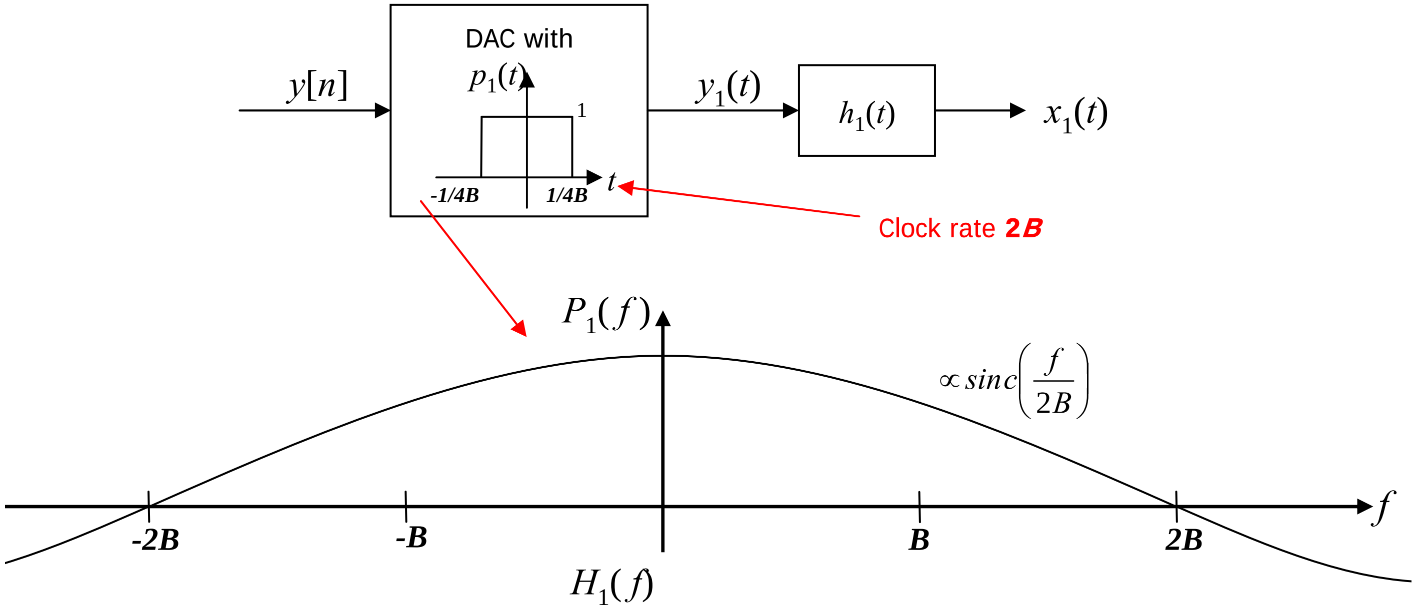


$$\propto \frac{1}{sinc\left(\frac{f}{2B}\right)}$$

D/A Conversion (cont.)

❖ Examples for Upsampling (cont.)

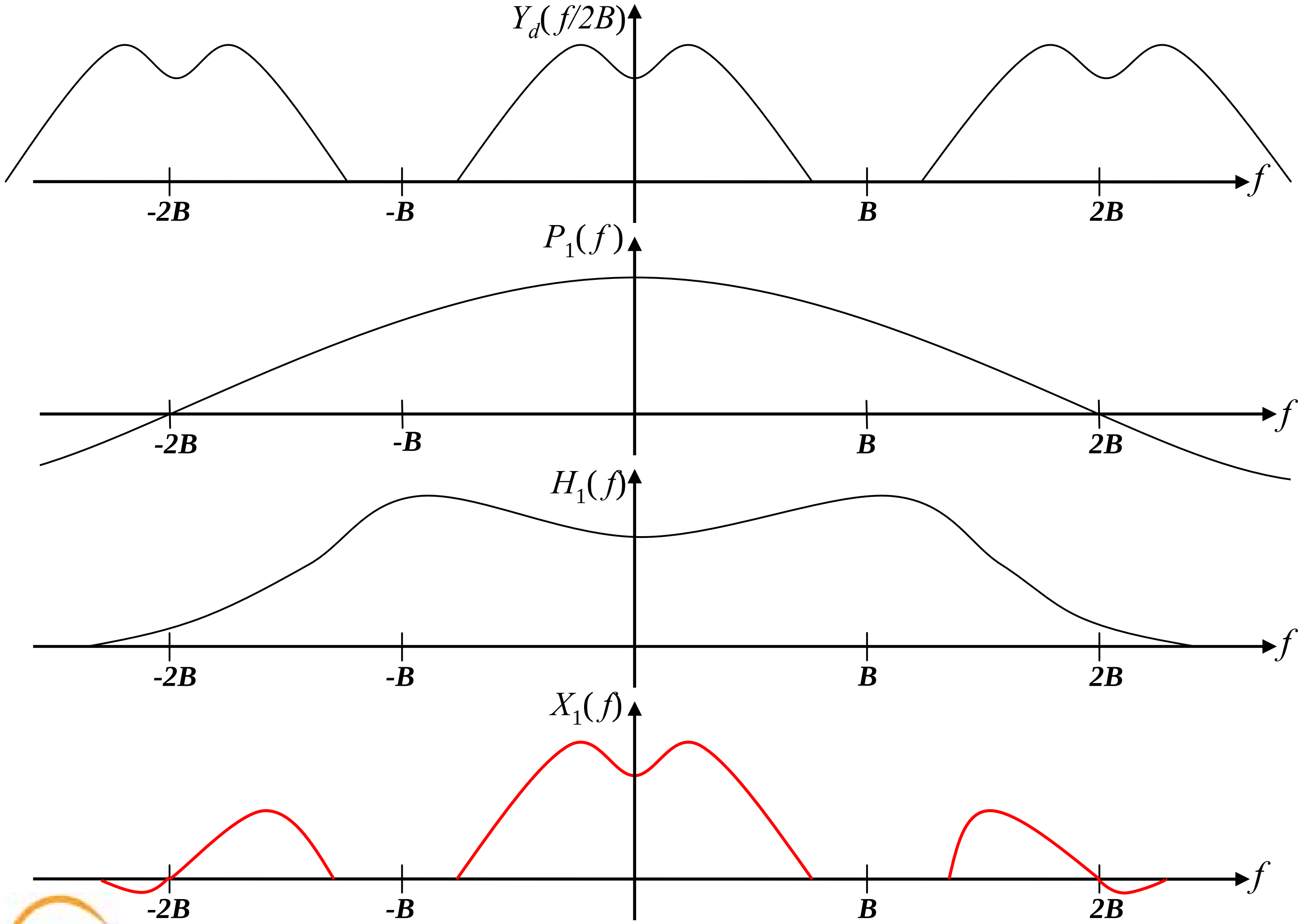
Q1. Sketch the output frequency response of $x_1(t)$



A. $X_1(f) = H_1(f) Y_1(f)$

$$= H_1(f) P_1(f) Y_d\left(\frac{f}{2B}\right)$$

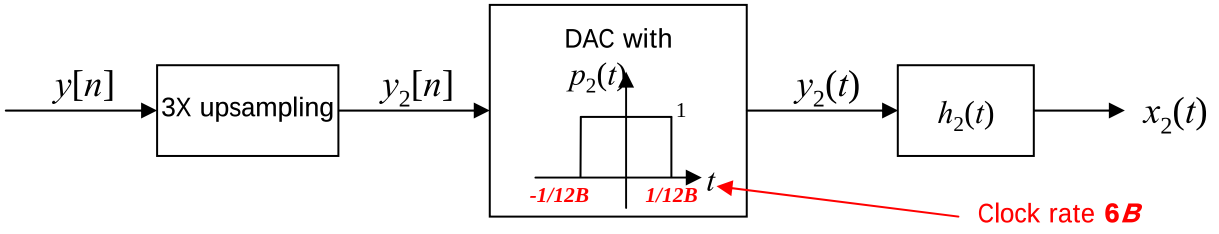
D/A Conversion (cont.)

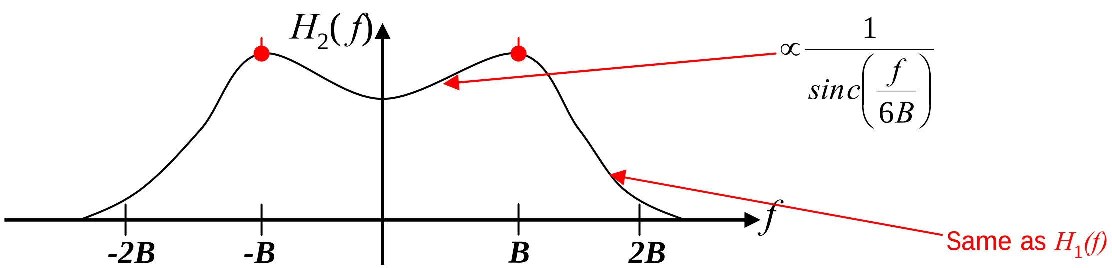


D/A Conversion (cont.)

❖ Examples for Upsampling (cont.)

Q2. Sketch the output frequency response of $x_2(t)$





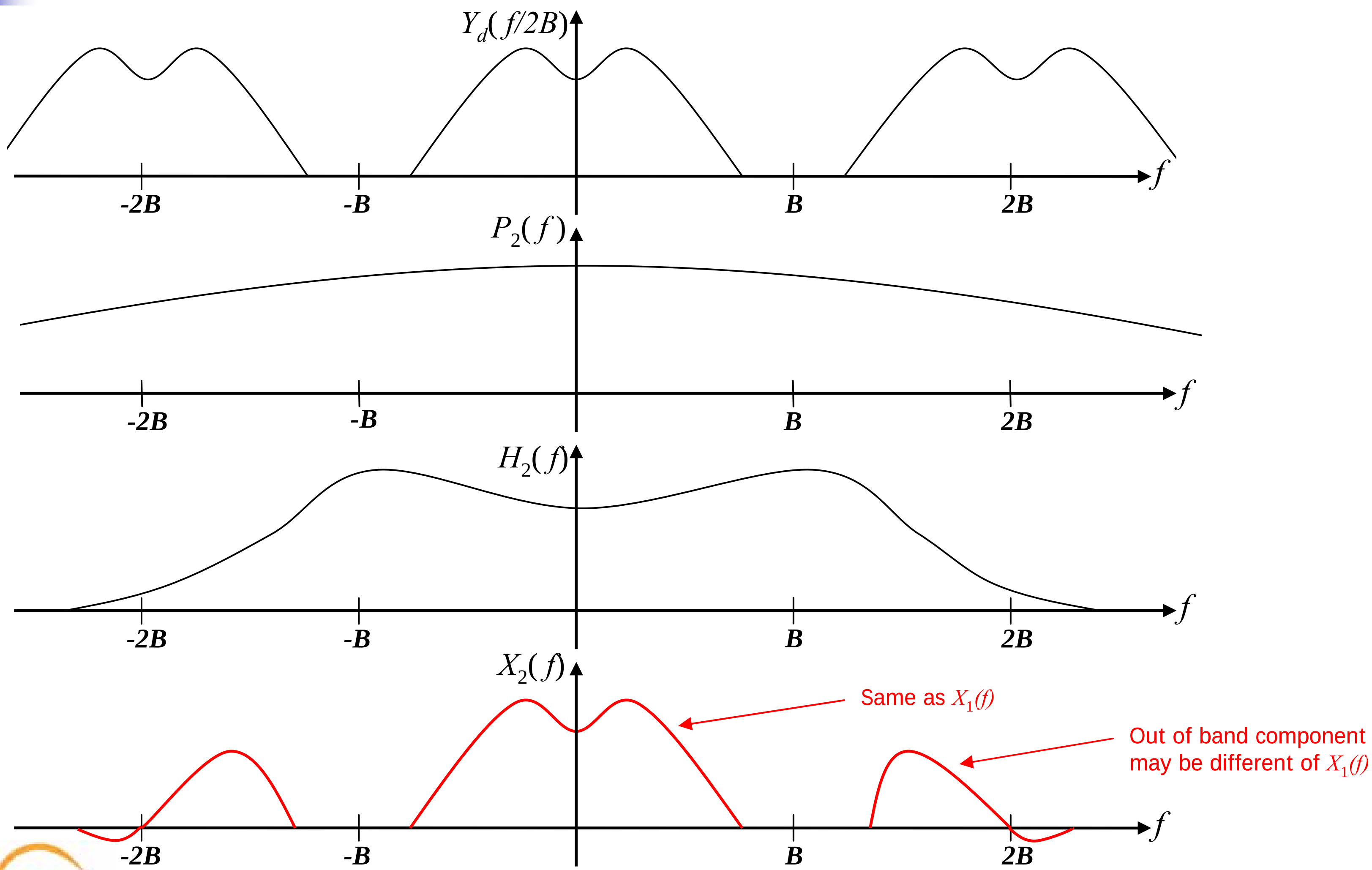
A. $X_2(f) = H_2(f)Y_{2,c}(f) = H_2(f)P_2(f)Y_{2,d}(f/6B)$

$$= H_2(f)P_2(f)Y_d(3f/6B)$$

$$= H_1(f)P_2(f)Y_d(f/2B)$$

$$= X_1(f), \quad |f| < B$$

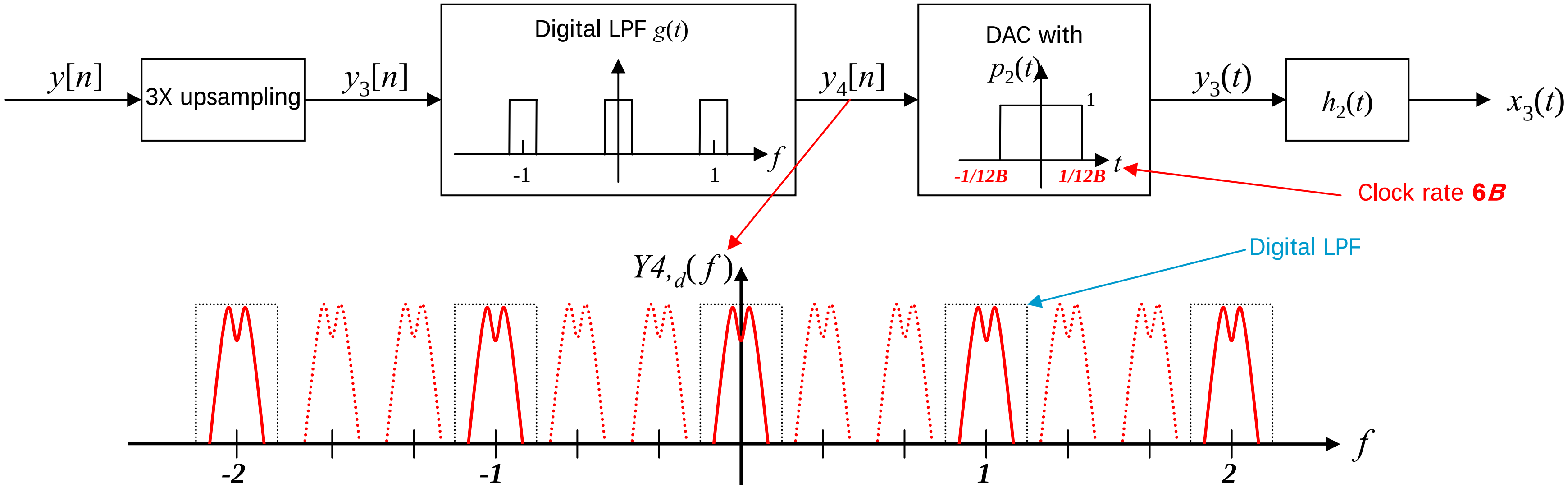
D/A Conversion (cont.)



D/A Conversion (cont.)

❖ Examples for Upsampling (cont.)

Q3. Sketch the output frequency response of $x_3(t)$



A. $X_3(f)=H_2(f)Y_{3,c}(f)=H_2(f)P_2(f)Y_{4,d}(f/6B)$

$$=H_2(f)P_2(f)G(f/6B)Y_3(f/6B)$$

$$=H_2(f)P_2(f)G(f/6B)Y_d(f/2B)$$

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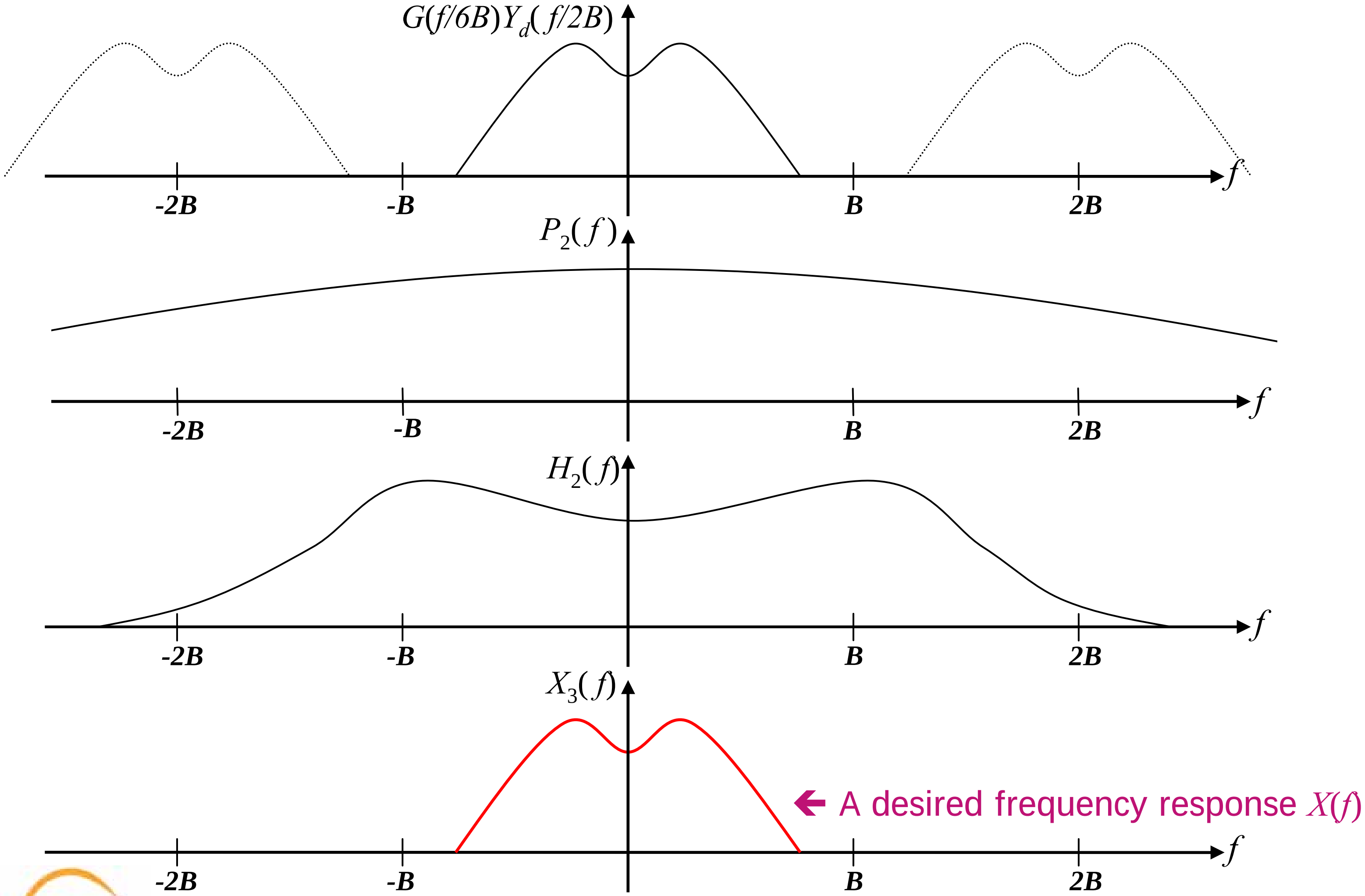
D/A Conversion (cont.)



D/A Conversion (cont.)

❖ Digital IF upconversion

We want to generate a real-values bandpass signal $x(t) = Re\{\sum_{n=-\infty}^{\infty} d[m]p(t-mT)e^{j2\pi f_c t}\}$

A1. Continuous-time Direct upconversion using D/A and I-Q modulation

➔ Phase noise, mismatch b/w I- and Q-branch

A2. Continuous-time IF upconversion

➔ Still, trouble with the imperfect balance b/w I- and Q-branch

A3. Digital IF upconversion

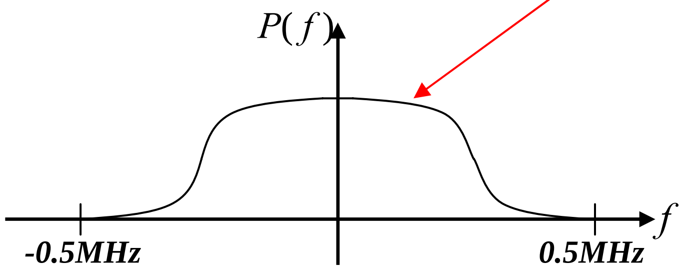
➔ Very high-rate DACs are now available.

D/A Conversion (cont.)

❖ Examples : Digital IF upconversion

We want to generate a continuous-time IF signal with bandwidth $1MHz$ by using a D/A with conversion-rate $200MHz$ and maximum output BW $100MHz$.

IF signal : $x(t) = Re\{ \sum_{n=-\infty}^{\infty} d[m] p(t-mT) e^{j2\pi f_{IF} t} \}, \quad f_{IF} \approx 100MHz$



Q1. Design a digital system whose output is fed to the DAC

A. We need to design a digital system that generate a sampled version of $x(t)$ at rate $200MHz$

Q2. Compared to the BW 1MHz of the signal, is $200MHz$ sampling rate too high?

A. Yes : sampled more than 100 times per symbol interval ➔ not necessary

D/A Conversion (cont.)

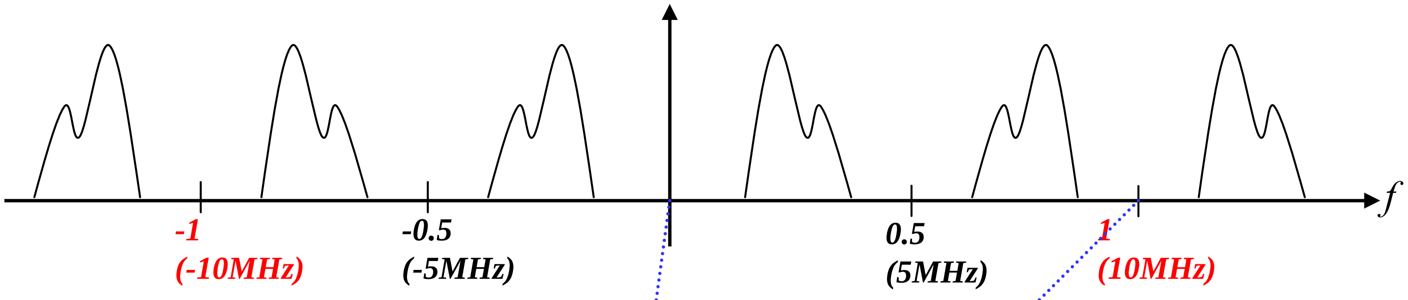
❖ Examples : Digital IF upconversion (cont.)

Q3. How can we reduce the complexity?

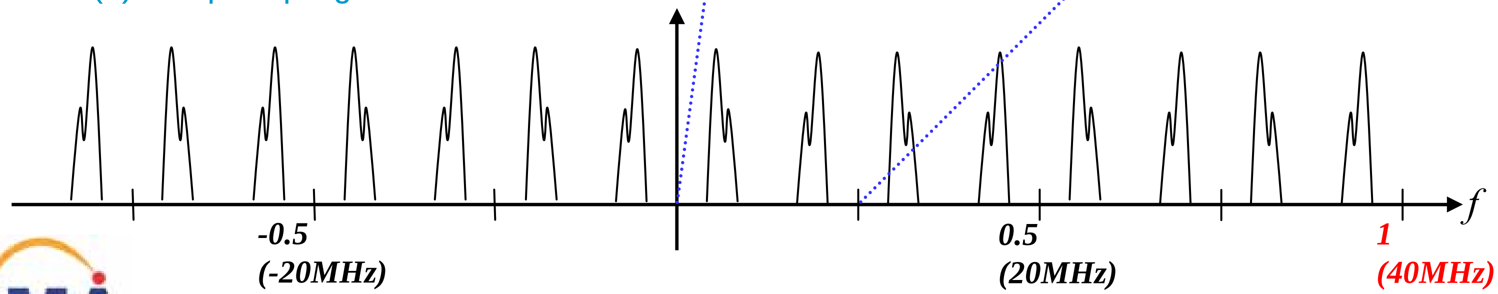
A. By upsampling and filtering followed by upsampling and filtering...

Assumption : IF converter that use 4X and 5X upsamplers and HPFs

(1) Generate the sampled version of $Re\{x_l(t)e^{j2\pi(7.5MHz)t}\}$ at rate $10MHz$.



(2) 4X upsampling



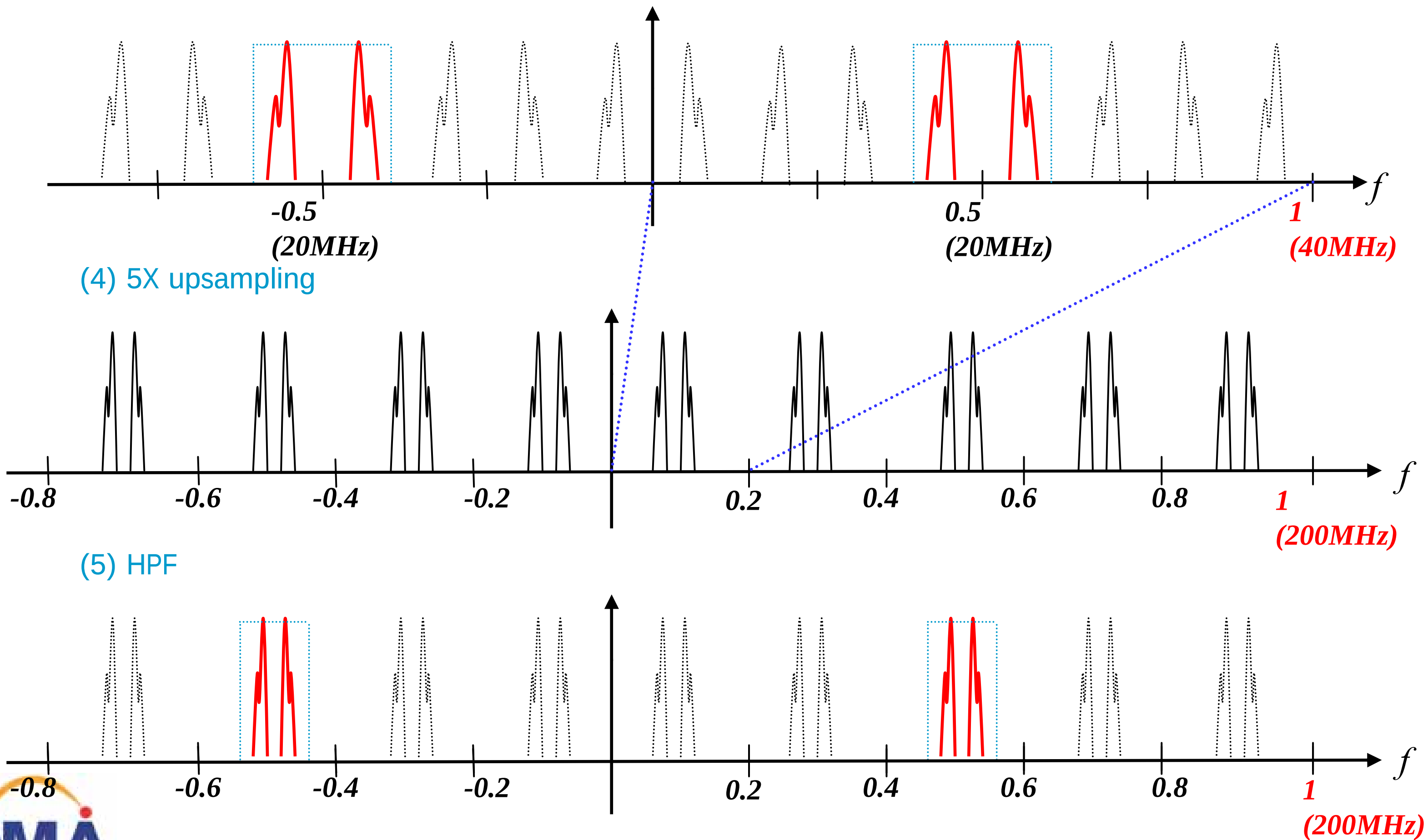
D/A Conversion (cont.)

❖ Examples : Digital IF upconversion (cont.)

(3) HPF

(4) 5X upsampling

(5) HPF



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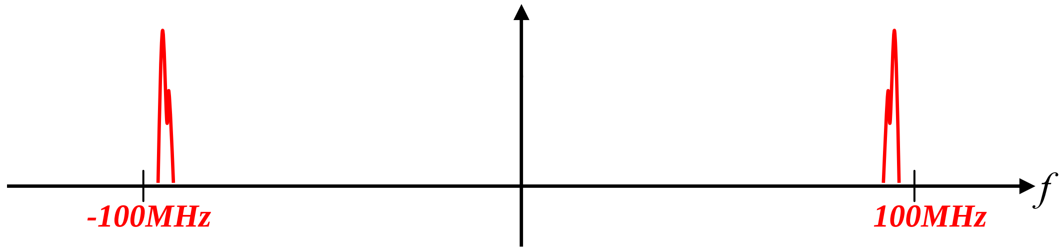
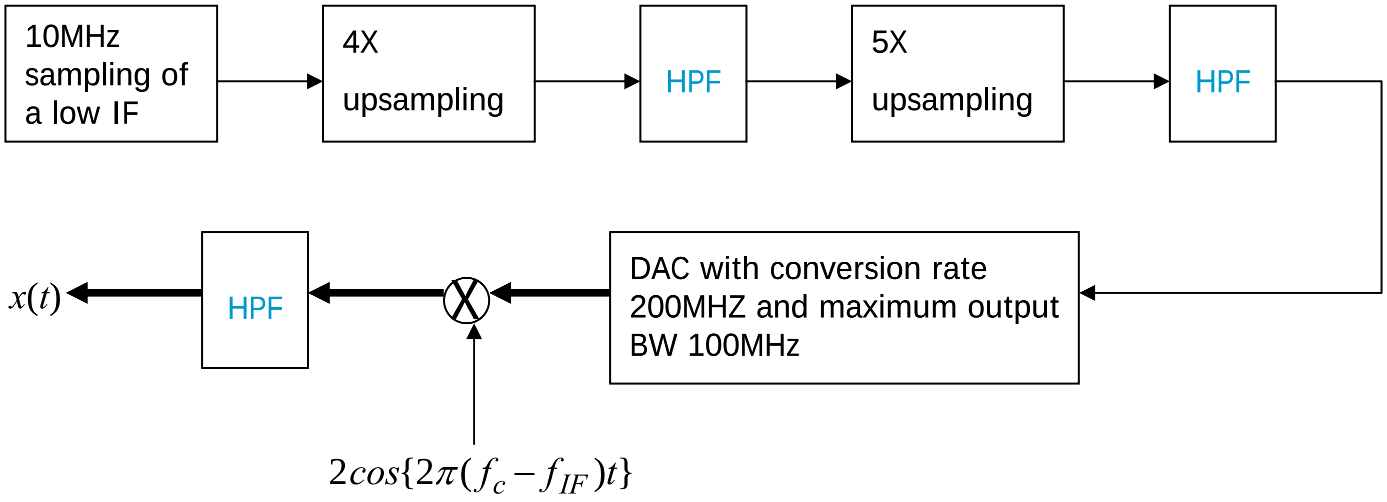
D/A Conversion (cont.)

❖ Examples : Digital IF upconversion (cont.)

(7) D/A conversion using DAC with conversion rate 200MHz and output BW 100MHz



Summary



Introduction to OFDM

❖ Characteristics of OFDM (Orthogonal Frequency division multiplexing)

- Parallel data transmission with very long symbol duration
  - Robust under multi-path channels
- Transformation of a frequency-selective channel into N frequency-flat channels
- Cyclic prefix (CP) is used to efficiently eliminate inter-symbol interference (ISI).
- High spectral efficiency of the orthogonal subcarriers
- Efficient implementation of the transmitter-receiver pair using Inverse Fast Fourier transform (IFFT) and Fast Fourier transform (FFT)
- High peak-to-average power ratio (PAPR) and sensitivity to carrier frequency offset (CFO)

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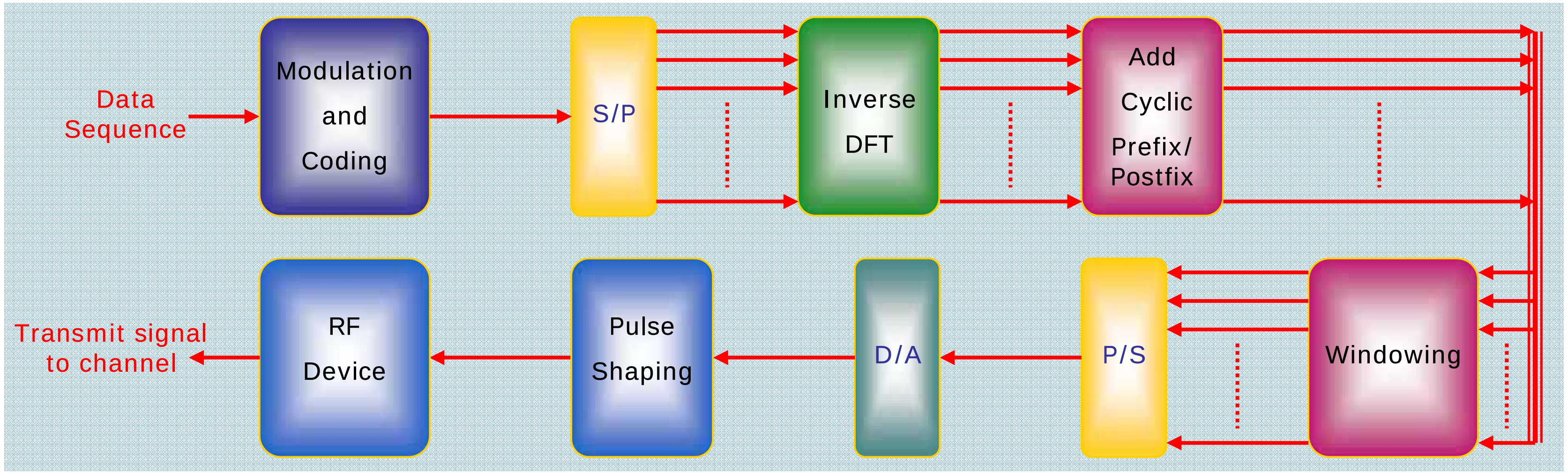
Introduction to OFDM (cont.)

❖ General system structure of OFDM transmitter



Review of Non-IDFT-Based Signal Model and its PSD

❖ **Non-IDFT-based** signal model for OFDM system

- Common continuous-time signal model in many literatures for **a single OFDM symbol in complex baseband** would be

$$Y(t)=\sum_{k=1}^{K} X_k e^{j2\pi f_k t}, \quad 0 \le t < T_0$$

where X_k is the data symbol transmitted by the kth subcarrier whose carrier frequency is f_k .

- The length of a single OFDM symbol and the subcarrier frequencies are related as

$$f_k - f_{k'} = \frac{k-k'}{T_0}$$

which leads to the orthogonality of the signals $e^{j2\pi f_k t}$ for $k=1,2,\ldots,K$ in the observation interval with time duration T_0 .

- For simplicity, assume that X_k's are uncorrelated, proper complex, zero-mean random variables with unit variance.

Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ **Non-IDFT-based** signal model for OFDM system (cont.)

- For the transmission of **more than one OFDM symbol**, the signal model is modified to

$$\hat{Y}(t)=\sum_{k=1}^{K}\left(\sum_{m=-\infty}^{\infty}X_k[m]w_C(t-mT_b)\right)e^{j2\pi f_k t}$$

which $X_k[m]$ is the kth subcarrier data symbol for time-index m, $1/T_b$ is the OFDM block transmission rate and $w_C(t)$ is the windowing function.

- T_b is the OFDM block transmission time and selected as larger than or equal to $T_0+\tau_{\max}$ where $\tau_{\max}$ is the maximum delay spread of the channel.

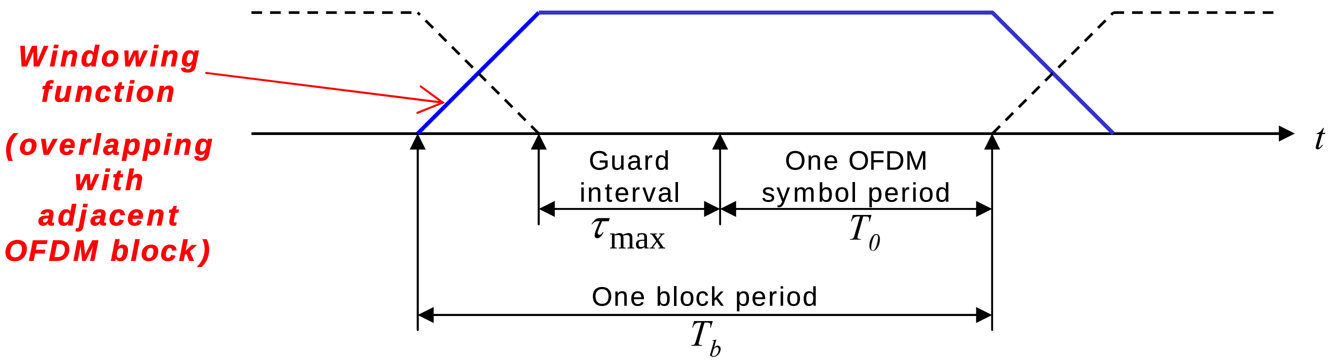
- The shape of the roll-off of the windowing function $w_C(t)$ is usually designed to reduce the inter-carrier interference (ICI) under carrier frequency offset (CFO).

  ➔ If no CFO, not only IBI but also ICI can be completely removed.

Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ Example of **non-IDFT based** continuous-time signal structure with windowing

$$\hat{Y}(t) = \sum_{k=1}^{K} \left(\sum_{m=-\infty}^{\infty} X_k[m] w_C(t - mT_b) \right) e^{j2\pi f_k t}$$



Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ Power spectral density (PSD) of **non-IDFT-based** signal model

- The PSD of this signal model is given by

$$S_{\hat{Y}\hat{Y}}(f) = \frac{1}{T_b} \sum_{k=1}^{K} \left| W_C(f - f_k) \right|^2$$

where $W_C(f)$ is the continuous-time Fourier transform (CTFT) of the windowing function $w_C(t)$.

- Note that $X_k[m]$'s are uncorrelated, proper complex, zero-mean random variables with unit variance.

- If a wideband analog filter is additionally used, if some subcarriers are null-out, and if different transmit power is allocated for each subcarrier, signal model can be rewritten to a more general form shown in the next page.

Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ Power spectral density (PSD) of **non-IDFT-based** signal model (cont.)

- General form of non-IDFT-based signal model

$$Z(t)=p(t)*\left[\sum_{k=1}^{K}\left(\sum_{m=-\infty}^{\infty}\alpha_k X_k[m]w_C(t-mT_b)\right)e^{j2\pi f_k t}\right]$$

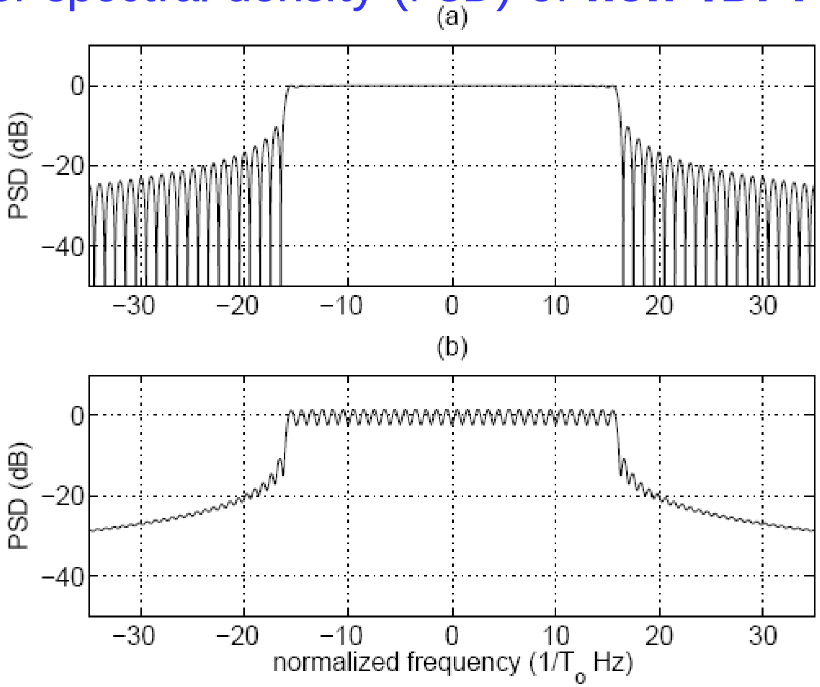
  - $p(t)$ is the complex envelope of the bandpass transmit filter.
  - α_k is the complex weighting factor for the kth subcarrier.
  (α_k is zero. ➔ The kth subcarrier is called a *null subcarrier*.)
  - $X_k[m]$ is the kth subcarrier data symbol for time-index m.
  - $w_C(t)$ is the continuous-time windowing function.
  - $1/T_b$ is the OFDM block transmission rate.

- The PSD of this signal model is given by

$$S_{ZZ}(f)=\frac{|P(f)|^2}{T_b}\sum_{k=1}^{K}|\alpha_k|^2|W_C(f-f_k)|^2.$$

Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ Power spectral density (PSD) of **non-IDFT-based** signal model (cont.)



- Parameter

  $\alpha_k = 1, \ \forall k$

  $K = 32$

- Rectangular windowing

  (a) : $T_b = T_0$

  (b) : $T_b = 1.25 T_0$

  Window duration= T_b

- (a) : Fourier transform of the rectangular window function is a sinc function.

  ➔ PSD is almost flat except at the edges.

  (At the edge, the PSD decays approximately at the rate $1/f^2$.)

- (b) : $\sum_{k=-\infty}^{\infty} |W_C(f - f_k)|^2$ is a periodic function with period $1/T_0$.

  ➔ PSD is not flat in the mid-bands.

Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ OFDM signal model in IEEE 801.11.a

- Baseband signal frame : $r_{PACKET}(t)=r_{PREAMBLE}(t)+r_{SIGNAL}(t-t_{SIGNAL})+r_{DATA}(t-t_{DATA})$

  - $t_{SIGNAL}=16\mu s$, $t_{Data}=20\mu s$

- Subframe : $r_{SUBFRAME}(t)=w_{T,SUBFRAME}(t)\sum_{k=-N_{ST}/2}^{N_{ST}/2} C_k e^{j2\pi k\Delta_f(t-T_{GUARD})}$

  - $w_{T,\,SUBFRAME}(t)$: Continuous-time windowing function with duration t_{TR}

  - C_k : Data, pilots, or training symbols

  - $\Delta_f=1/T_{FFT}$: Subcarrier frequency spacing (=$1/T_0$ in our models)

  - T_{GUARD} : Guard time to create the cyclic prefix

    ➔ For the short training seq. (=$0\mu s$), long training seq. (=T_{GI2}), data OFDM symbols (=T_{GI})

- Windowing function may have the time duration extended over than one OFDM period, T_{FFT}, and the transition time overlapped via an adjacent OFDM symbol windowing. (Also in our signal model)

Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ OFDM signal model in IEEE 801.11.a (cont.)

- Timing-related parameter

Parameter	Value
N_{SD} : Number of data subcarriers	48
N_{SP} : Number of pilots subcarriers	4
N_{ST} : Number of total subcarriers	52 $(N_{SD}+N_{SP})$
Δ_f: Subcarrier frequency spacing	0.3125 MHz (=20 MHz/64)
T_{FFT} : IFFT/FFT period	3.2 μs $(1/\Delta_f)$
$T_{PREAMBLE}$: PLCP Preamble duration	16 μs $(T_{SHORT}+T_{LONG})$
T_{SIGNAL} : Duration of the SIGNAL BPSK-OFDM symbol	4.0 μs $(T_{GI}+T_{FFT})$
T_{GI} : GI duration	0.8 μs $(T_{FFT}/4)$
T_{GI2} : Training symbol GI duration	1.6 μs $(T_{FFT}/2)$
T_{SYM} : Symbol interval	4 μs $(T_{GI}+T_{FFT})$
T_{SHORT} : Short training sequence duration	8 μs $(10T_{FFT}/4)$
T_{LONG} : Long training sequence duraion	8 μs $(T_{GI2}+2T_{FFT})$

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Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ OFDM signal model in IEEE 801.16.e and WiBro

- OFDM transmit signal

$$s(t) = \mathrm{Re}\left\{ \sum_{\substack{k=-N_{ST}/2 \\ k \neq 0}}^{N_{ST}/2} C_k e^{j2\pi k \Delta_f (t-t_g)} \right\}, 0 \le t \le T_s$$

- C_k : Complex number of data symbol (QAM)
- $\Delta_f = F_s / N_{FFT}$: Subcarrier frequency spacing
- N_{FFT} : Smallest power by 2 greater than number of used subcarriers
- F_s : Smallest power by 2 greater than number of used subcarriers
- T_g : Guard time to create the cyclic prefix

IDFT-Based Signal Model and its PSD

❖ 1. **IDFT-based discrete-time** signal model

- Problem of the non-IDFT signal model : Its straightforward implementation requires $|\kappa|$ local oscillators that are tuned to $|\kappa|$ different subcarrier frequencies where $\kappa = \{k : \alpha_k \neq 0\}$, i.e., $|\kappa|$ is the number of non-null subcarriers.

  ➔ Idea : Using the K times sampling of the signal $e^{j2\pi f_k t}$ in the interval $0 \le t < T_0$ at the rate K/T_0 for $k=1,2,\ldots,K$

  ➔ Orthogonal vectors that are proportional to the column vectors of the K-point IDFT matrix

- The most common discrete-time signal model for a single vector of OFDM symbols is

$$\underline{Y} = S\,\underline{X}$$

where $\boldsymbol{S}$ is the K-point IDFT matrix whose (k, l)th entry is given by $s_k[l] = \frac{1}{\sqrt{K}} e^{j\frac{2\pi kl}{K}}$, $\underline{X}$ is the vector consisting of K data symbols $\{X_k\}_{k=1}^{K}$ and $\underline{Y}$ is the vector of OFDM symbol.

IDFT-Based Signal Model and its PSD (cont.)

❖ 1. **IDFT-based discrete-time** signal model (cont.)

- This procedure can be rewritten as

$$\underline{Y} = S\underline{X} = X_1 \begin{bmatrix} \frac{1}{\sqrt{K}} e^{j\frac{2\pi 1\cdot 1}{K}} \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi 2\cdot 1}{K}} \\ \vdots \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi K\cdot 1}{K}} \end{bmatrix} + X_2 \begin{bmatrix} \frac{1}{\sqrt{K}} e^{j\frac{2\pi 1\cdot 2}{K}} \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi 2\cdot 2}{K}} \\ \vdots \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi K\cdot 2}{K}} \end{bmatrix} + \cdots + X_K \begin{bmatrix} \frac{1}{\sqrt{K}} e^{j\frac{2\pi 1\cdot K}{K}} \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi 2\cdot K}{K}} \\ \vdots \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi K\cdot K}{K}} \end{bmatrix}$$

- Operation characteristics

  - The data symbols $\{X_k\}_{k=1}^{K}$ are orthogonally multiplexed.

  - The energy of different data symbol is centered at different subcarrier frequencies.

IDFT-Based Signal Model and its PSD (cont.)

❖ 1. **IDFT-based discrete-time** signal model (cont.)

- For a transmission over a continuous-time channel, the OFDM symbol is D/A-converted then up-converted using a local oscillator whose frequency is tuned to the center frequency of the transmitted bandpass signal.

  ➔ performed efficiently without multiple local oscillators

- In many case, IDFT multiplication can be implemented by a computationally efficient inverse Fast Fourier Transform (IFFT) instead of IDFT, then multiplication requires only $O(N\log N)$ operations reduced from $O(N^2)$

- Not for a single OFDM symbol but for a sequence of OFDM symbols, what is the signal model considering all of analog wideband filter, D/A converter, digital interpolation filter, cyclic prefix (or postfix), windowing sequence?

  ➔ Needs some modifications to IDFT-based continuous-time signal model

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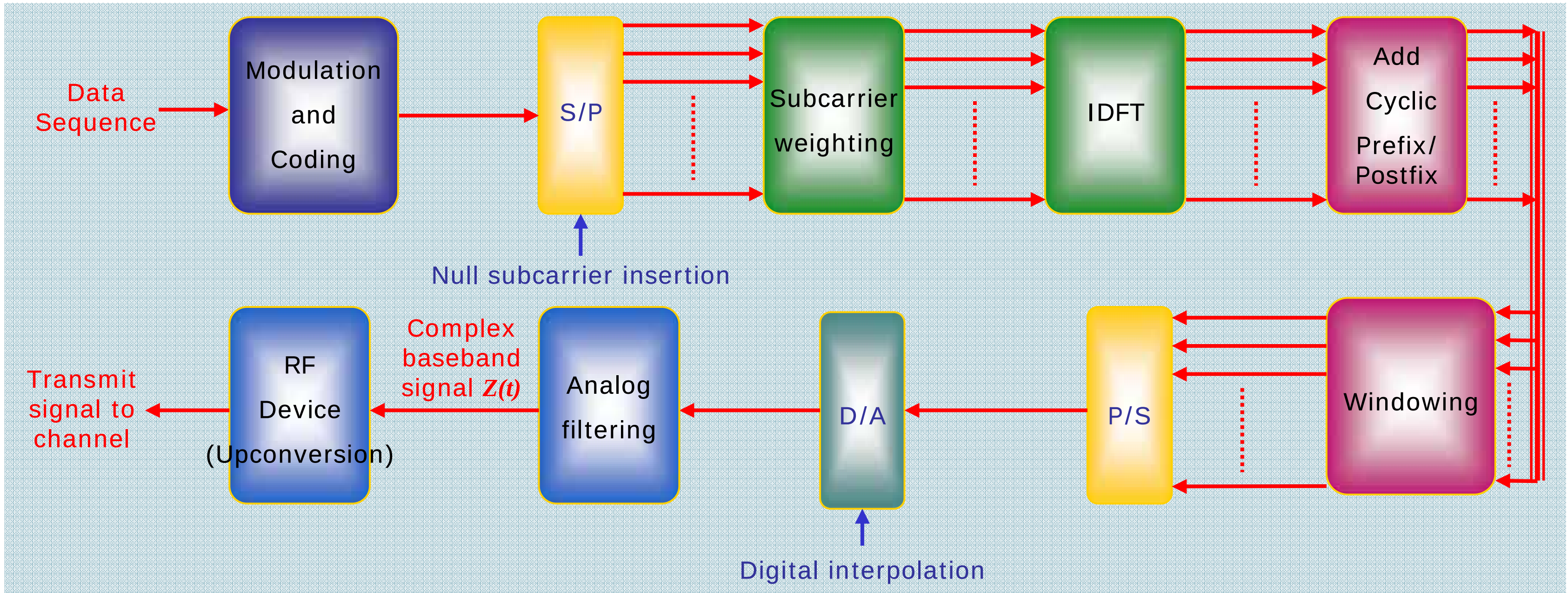
IDFT-Based Signal Model and its PSD (cont.)

❖ 2. **IDFT-based continuous-time** signal model

- Revised system structure for IDFT-based transmitter block diagram for OFDM



- IDFT matrix point : K

- Length of cyclic prefix : $L+M$, cyclic postfix : M

($L \geq$ channel length, M : # of overlapped entries of the vectors after P/S)

- Length of windowing sequence : $K+L+2M$

IDFT-Based Signal Model and its PSD (cont.)

❖ 2. **IDFT-based continuous-time** signal model (cont.)

- *D/A converter* : would be functions such as the sample-and-hold

  - If the D/A input is clocked in every T_c [sec], then D/A converter can be viewed as a linear modulator employing a rectangular pulse with duration T_c.

  ➔ The D/A converter combined with the analog filter can be viewed as a linear modulator employing a pulse, say $p(t)$ with the pulse transmission rate $1/T_c$ (also when D/A uses a digital interpolation that operates oversampling).

- *Windowing sequence* : Usually designed to properly weight the first M and the last M symbols,

  ➔ The proper shape of the roll-off of the windowing sequence reduces the inter-channel interference (ICI).

- *Null subcarriers* : Not to use an analog filter with a sharp transition from passband to stopband, some carriers whose the index k's are around $k \approx K/2$ being nulled out.

  - The useful subcarriers will be less than K, but still K-point IDFT is performed with data symbols.

IDFT-Based Signal Model and its PSD (cont.)

❖ 2. **IDFT-based continuous-time** signal model (cont.)

- General form of the complex baseband equivalent for IDFT-based continuous-time signal model is,

$$Z(t) = \sum_{k=1}^{K} \sum_{m=-\infty}^{\infty} \alpha_k X_k[m] s_k(t - mT_b).$$

- α_k is the subcarrier weighting factor.

- $X_k[m]$ is the mth symbol for the kth subcarrier.

- T_b is the OFDM symol block period given by $T_b = (K + L + M)T_c$.

- T_c is the transmit pulse time duration.

(OFDM subcarrier spacing is $1/T_0$ [Hz] , so $KT_c = T_0$)

- $s_k(t)$ is the kth subcarrier waveform.

IDFT-Based Signal Model and its PSD (cont.)

❖ 2. **IDFT-based continuous-time** signal model (cont.)

- $s_k(t)$ is defined by considering the windowing sequence $w_D[l]$, IDFT matrix element $s_k[l]$, and transmit pulse $p(t)$. So, $s_k(t)$ is given by

$$s_k(t) = \sum_{l=1}^{K+L+2M} \alpha_k[l]\, p(t - lT_c)$$

where $\alpha_k[l] = w_D[l] s_k[l]$ for $k = 1,2,3,\ldots,K$, and $l = 1,2,3,\ldots,K+L+2M$.
($s_k[l]$ is from IDFT matrix (k, l)th element and is defined as $s_k[l] = \frac{1}{\sqrt{K}} e^{j\frac{2\pi kl}{K}}$, but now up to $l = K+L+2M$)

- $p(t)$ is the equivalent transmit pulse including all the effects of the D/A converter, digital interpolation filter, and analog wideband filter.

- This modified signal model looks similar to a symbol synchronous DS-CDMA signal model where $s_k(t)$ serves as the signature waveform and $a_k[l]$ serves as the signature sequence, both the kth users.

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IDFT-Based Signal Model and its PSD (cont.)

❖ Power spectral density (PSD) of **IDFT-based continuous** signal model

- Fourier transform of the kth subcarrier waveform $s_k(t)$

$$S_k(f)=\frac{1}{\sqrt{K}}P(f)\sum_{k=1}^{K+L+2M} w_D[l]\exp\left(-j2\pi\left(f-\frac{k}{T_0}\right)\right)=\frac{1}{\sqrt{K}}P(f)W_D\left(f-\frac{k}{T_0}\right)$$

- Using above result, PSD of this signal model is obtained as

$$S_{ZZ}(f)=\frac{\left|P(f)\right|^2}{(K+L+M)T_0}\sum_{k=1}^{K}\left|\alpha_k\right|^2\left|W_D(f-\frac{k}{T_0})\right|^2$$

where $W_D(f)$ is the discrete-time Fourier transform of the windowing sequence $\{w_D[l]\}_{l=1}^{K+L+2M}$ defined as $W_D(f)=\sum_{l=1}^{K+L+2M} w_D[l]e^{-j2\pi f T_c l}$.

IDFT-Based Signal Model and its PSD (cont.)

❖ Power spectral density (PSD) of **IDFT-based continuous** signal model (cont.)

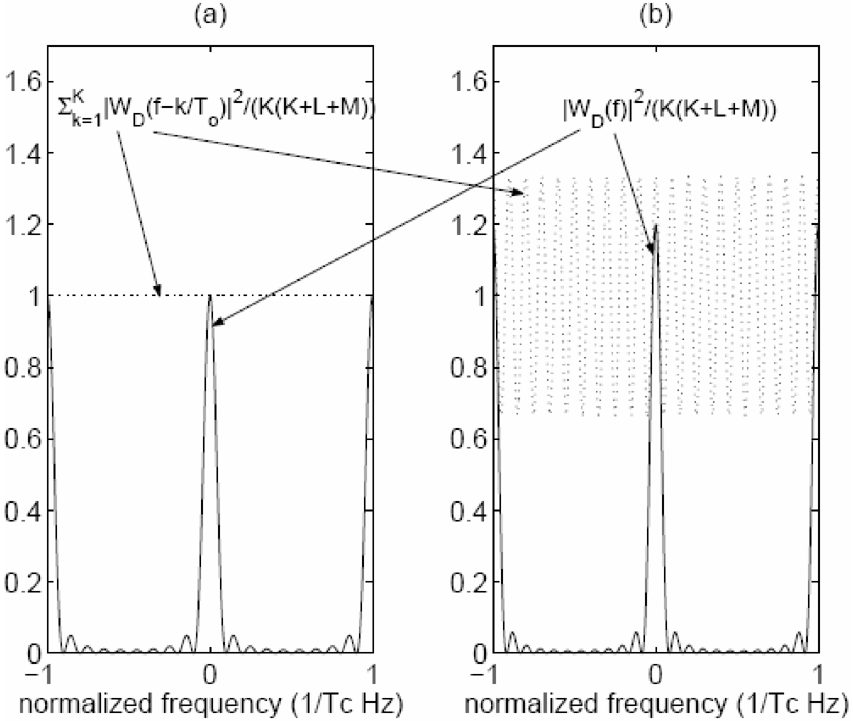
Non-IDFT-based : $S_{ZZ}(f)=\frac{|P(f)|^2}{T_b}\sum_{k=1}^{K}|\alpha_k|^2|W_C(f-f_k)|^2$

IDFT-based : $S_{ZZ}(f)=\frac{|P(f)|^2}{(K+L+M)T_0}\sum_{k=1}^{K}|\alpha_k|^2\left|W_D(f-\frac{k}{T_0})\right|^2$

- Significant difference from the non-IDFT-based model is that the Fourier transform of the windowing sequence $\{w_D[l]\}_{l=1}^{K+L+2M}$ is used instead of that of the continuous windowing function $w_c(t)$
- [Note] Two properties of $W_D(f)$
  - (1) $W_D(f)$ is periodic with a period $1/T_c$
  - (2) $W_D(f-k/T_0)$ is periodic in k with a period K under the assumption $KT_c=T_0$

IDFT-Based Signal Model and its PSD (cont.)

❖ Example of PSD of IDFT-based continuous signal model



- Parameter

Rectangular windowing sequence for

(a) : $K=10, L+M=0$

(b) : $K=10, L+M=2$

- [Note] $|W_D(f)|^2 = \left|\dfrac{\sin(\pi f(K+L+2M)T_c)}{\sin(\pi f T_c)}\right|^2$

- Increase in (K+L+2M) ➔ Decrease in the width of the main lobe and increase in the amount of in-band energy contained in the interval $-\dfrac{1}{2T_0} \le f \le \dfrac{1}{2T_0}$
- Periodic with period $1/T_c$

IDFT-Based Signal Model and its PSD (cont.)

- New insight into PSD of OFDM signals
  - Definition

    If the aggregate transmit pulse $p(t)$ has a positive excess bandwidth, then, due to the periodicity of $W_D(f)$, a logical subcarrier can have the signal energy concentrated at $f = \frac{k + lK}{T_0}$ for some integers l, that is, a logical subcarrier can have more than one physical subcarriers.

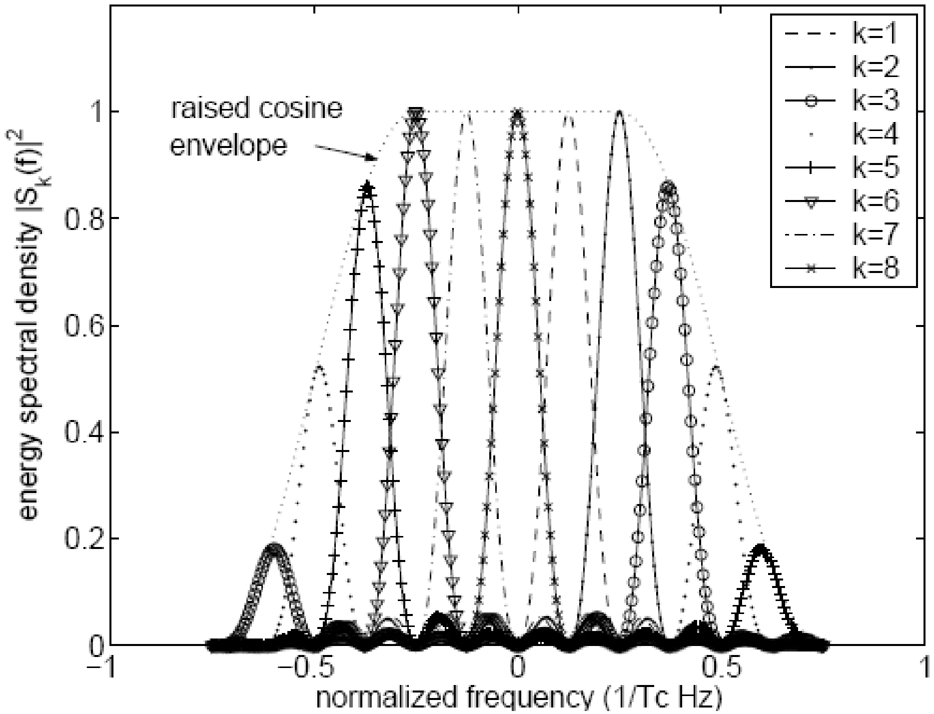
    → *Nominal subcarrier* : Physical subcarrier with the greatest energy, and

    Ghost subcarriers : Other physical subcarriers except the *nominal subcarrier*

  - For some OFDM logical subcarriers, we may observe two physical subcarriers, the nominal subcarriers and ghost carrier, using IDFT-based continuous signal model.

IDFT-Based Signal Model and its PSD (cont.)

❖ Example : Energy spectral density of the subcarrier waveforms



- $p(t)$ is the SRRC pulse with β=0.5, K=8, L=0, M=0, *no null subcarrier*
- The subcarriers with indexes k=3,4, and 5 have the significant portion of the energy split into two physical subcarriers located $1/T_c$ [Hz] apart.

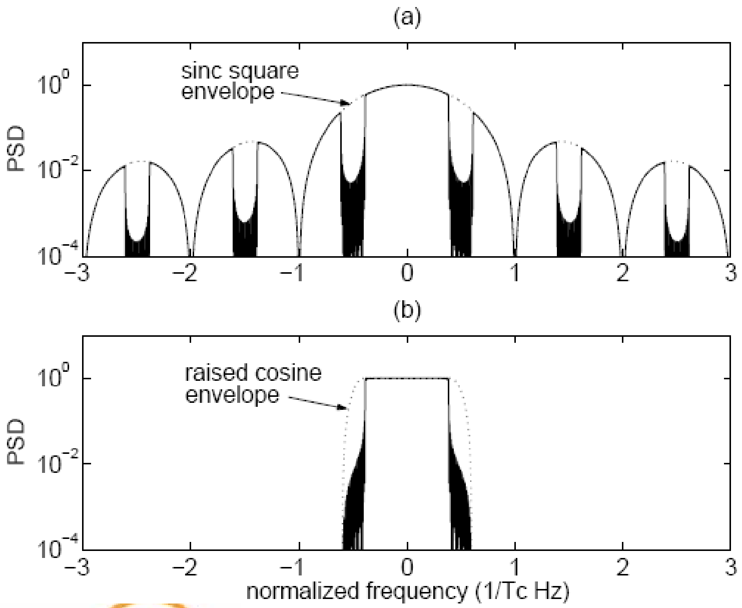
IDFT-Based Signal Model and its PSD (cont.)

❖ Example : Virtual roll-off effect by using proper null-subcarrier

- Very fast roll-off in the PSD may be needed, but designing such a filter is costly.

  ➔ Because we need a high-rate D/A converter, high order wideband transmit filter

- Efficient solution : By nulling subcarriers with indexes $k \approx K/2$

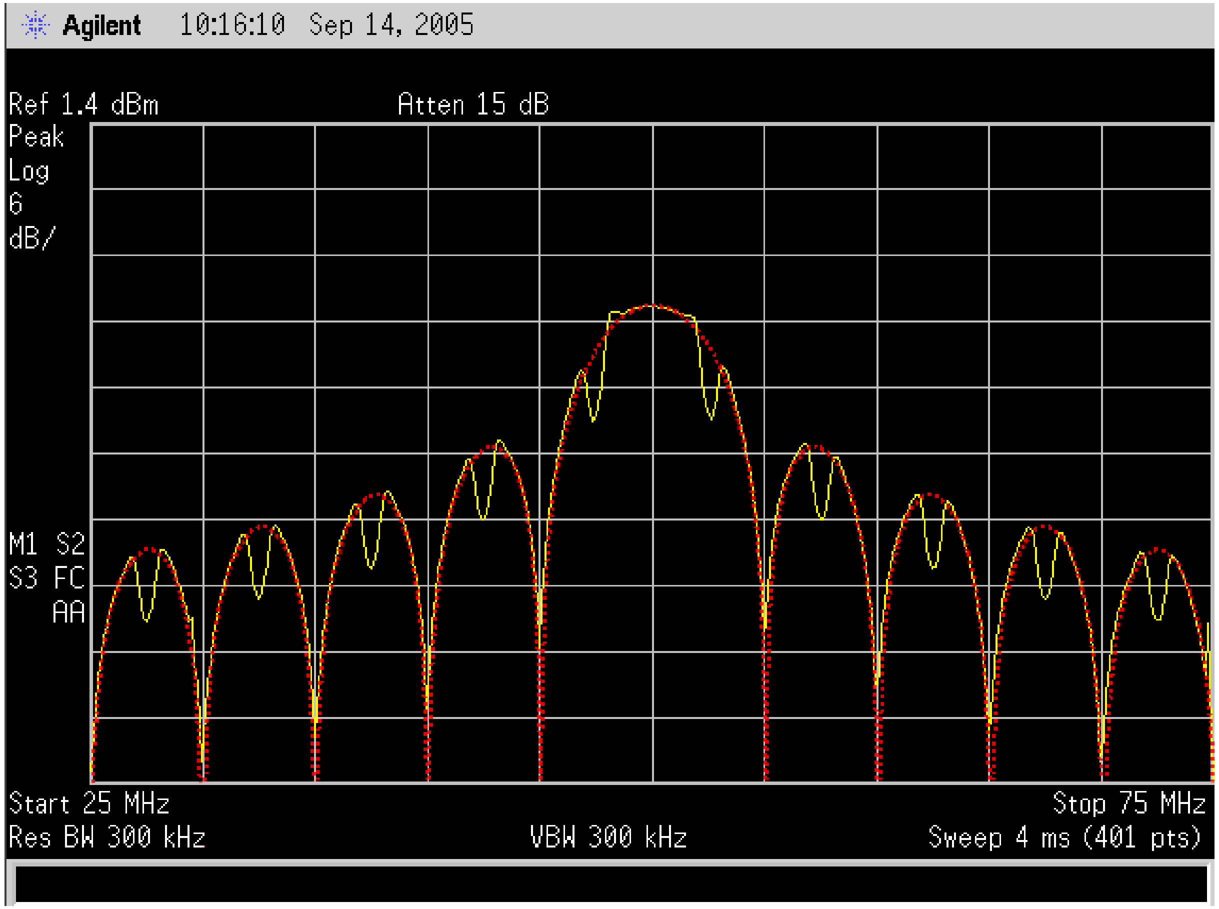


- PSD $S_{ZZ}(f)$ of complex baseband signal $Z(t)$
- Parameter
  - *K=128, L=0, M=0, 29 null-subcarriers*
  - *Sample-and-hold in D/A converter*
- (a) : No wideband analog filtering

  (b) : Use of an analog filter to make p(t) the SRRC pulse with β=0.2

➔ Possible to make a transition band very small

IDFT-Based Signal Model and its PSD (cont.)



IDFT-Based Signal Model and its PSD (cont.)

❖ In WiBro standard, the signal model is given by

$$s(t) = \mathrm{Re}\left\{ \sum_{\substack{k=-N_{ST}/2 \\ k \neq 0}}^{N_{ST}/2} C_k e^{j2\pi k \Delta_f (t-t_g)} \right\}, 0 \le t \le T_s$$

- A rectangular windowing function without any window overlapping is used.
- Non-IDFT-based signal model is used.

❖ To use IFFT in the transmitter, we need the following or an equivalent :

- A rectangular windowing sequence without any window overlapping
- A sample-and-hold D/A converter with inverse-sinc shaped transmit filter in the frequency band for non-null subcarriers

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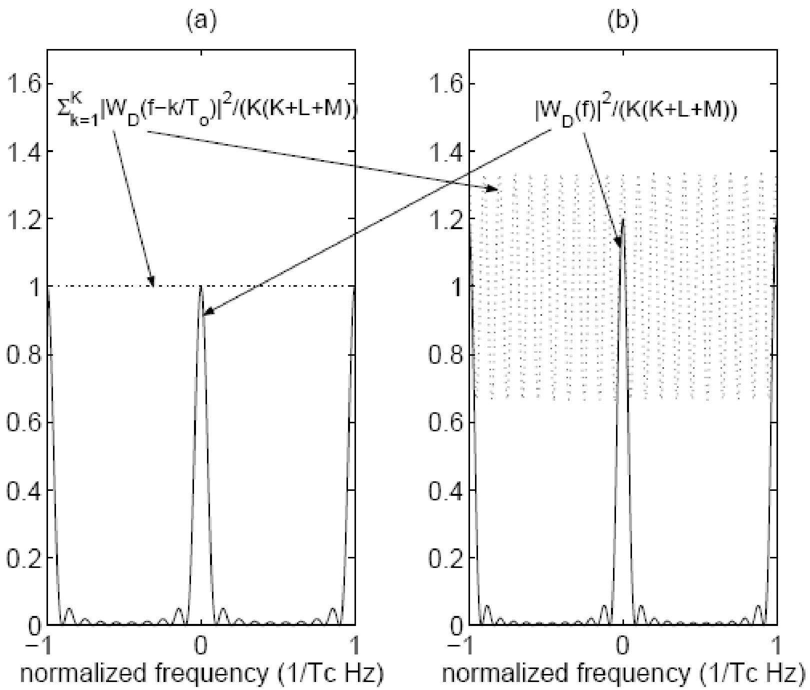
IDFT-Based Signal Model and its PSD (cont.)

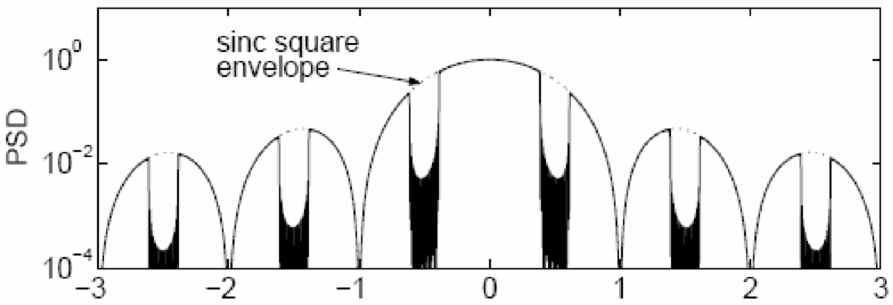


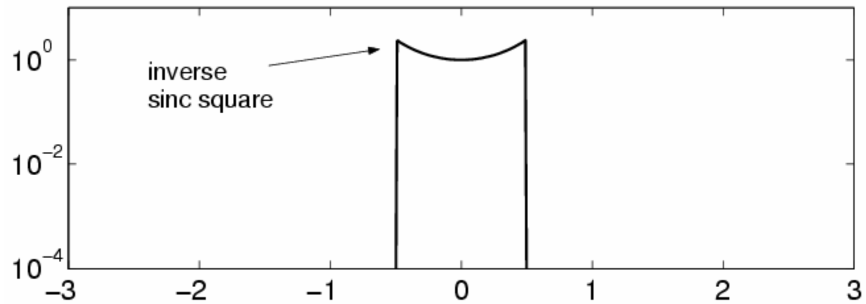
$$|W_D(f)|^2 = \left|\frac{\sin(\pi f(K+L)T_c)}{\sin(\pi f T_c)}\right|^2$$

$$\approx \left|\frac{\sin(\pi f(K+L)T_c)}{\pi f T_c}\right|^2$$

for *large K+L and*
for $f \approx 0$

IDFT-Based Signal Model and its PSD (cont.)





- The transmit filter must invert the sinc square envelope to equally distribute the power among non-null subcarriers.

- The transition from the passband to stopband must lie over the null-subcarrier frequency band.

Conclusions

❖ D/A conversion is a necessary procedure in modern digital audio systems and in digital communication systems.

$$\boxed{Y_c(f) = P(f) X_d(fT)}$$

❖ Upsampling combined with interpolation is often used in commercial DACs and in modern radio transceiver.

$$\boxed{Y_d(f) = X_d(fN)}$$

❖ IFFT-based OFDM signaling also requires D/A conversion.

$$\boxed{S_{ZZ}(f) = \frac{|P(f)|^2}{(K+L+M)T_0} \sum_{k=1}^{K} |\alpha_k|^2 \left| W_D\left(f - \frac{k}{T_0}\right) \right|^2}$$

- To ease the burden of designing a good interpolation filter, null subcarriers are used.
- The interpolation filter must have the transition from passband to stopband in the null subcarriers' frequency band.